

# Outward Accessibility in Urban Street Networks: Characterization and Improvements

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The dynamics of transportation through towns and cities is strongly affected by the topology of the connections and routes. The current work describes an approach combining complex networks and self-avoiding random walk dynamics in order to quantify in objective and accurate manner, along a range of spatial scales, the accessibility of places in towns and cities. The transition probabilities are estimated for several lengths of the walks and used to calculate the outward accessibility of each node. The potential of the methodology is illustrated with respect to the characterization and improvements of the accessibility of the town of São Carlos.

The intrinsic relationship between structure and dynamics seems to scaffold many dynamical processes in nature, from the flight of birds to the binding of proteins. Because of its inherent ability to represent and model the most diverse types of discrete structures, complex networks have received growing attention. Having initially focused attention on the characterization and modeling of the topology of interconnectivities (e.g. [1, 2, 3]), complex network research progressed steadily to encompass the relationship between structure and dynamics in the most diverse systems (e.g. [2, 4]). Though the connectivity does not completely define dynamics, it strongly affects it. This has become clear through investigations of relationships between structure and several types of dynamics, including diffusion (e.g. [5]), synchronization (e.g. [6]) and neuronal networks [7, 8]. Particularly, when the dynamics is modeled in terms of random walks (e.g. [5, 9]), the displacements of the respective moving agents are strongly influenced by several topological factors such as the number of connections at each node and the shortest path lengths between nodes. This type of stochastic dynamics presents an intrinsic potential for modeling the displacement of people within towns or cities. However, because traditional linear random walks allow a moving agent to visit edges and nodes more than once, implying null average displacement in the long term, it becomes important to consider more purposive types of displacements. Self-avoiding walks (e.g. [10, 11, 12, 13]) represent a natural simplified choice for modeling urban displacements, implying the agents to move away from their initial position in a more effective way while not repeating edges or nodes.

Complex networks have been used to characterize important topological, dynamical and spatial properties of cities (e.g., [14, 15, 16, 17, 18]). One important practical application of the structure-dynamics relationship concerns the characterization, modeling and planning of urban displacements. In a previous study, Rosvall et al. [14] considered shortest path lengths in order to quantify and compare the information needed to locate specific addresses in different cities. The current work applies the recently introduced concept of *accessibility* [19] in order to quantify in an objective and comprehensive way the outward accessibilities of each node of a town

(i.e. intersection or beginning of routes). The methodology is illustrated with respect to the specific application to the Brazilian town of São Carlos. Image processing and analysis methods were used to transform the plan of the town into a respective geographical planar network, where the nodes represent the crossings and the beginning of routes, while the edges correspond to the streets. Figure 1 shows the network derived from the central part of the town. Then, by simulating a series of self-avoiding walks starting at all nodes, transition probabilities from one node to another were estimated with respect to varying lengths of the walks. The *outward accessibility* of each node — expressing the diversity of routes between those points as well as the potential of the moving agent to visit a set of nodes in the shortest time — is quantified in terms of the entropy of the obtained transition probabilities, so that values close to 1 indicate maximum outward accessibility. One important property of the outward accessibility measurement is that self-avoiding walks initiating from nodes characterized by high accessibility for a given path length tend to visit all reachable nodes at that length in the shortest period of time. In addition, the outward accessibility intrinsically considers the number of alternative routes from the initial node to the reachable nodes. Nodes with high outward accessibility therefore have more balanced number of routes leading to the reachable nodes [19, 20].

We start by presenting the basic concepts about network representation. An unweighted and undirected complex network can be represented by a matrix  $K$ , called *adjacency matrix*. The dimension of this matrix for a network with  $N$  nodes and  $E$  edges is  $N \times N$ . If the nodes  $i$  and  $j$  are connected through an edge, the elements  $K(i, j)$  and  $K(j, i)$  of the adjacency matrix are set to 1; otherwise  $K(i, j) = K(j, i) = 0$ . Two nodes of the network are said to be adjacent if they share one edge. Two edges of the network are said to be adjacent if one extremity of each edge share the same node. The *degree* of a node  $i$  is the number of its immediate neighbors. A *walk* over the network is composed by a sequence of adjacent nodes, starting from an initial node and proceeding through successive steps  $h$ . A *self-avoiding walk* is a walk where the nodes and edges do not appear more than once.

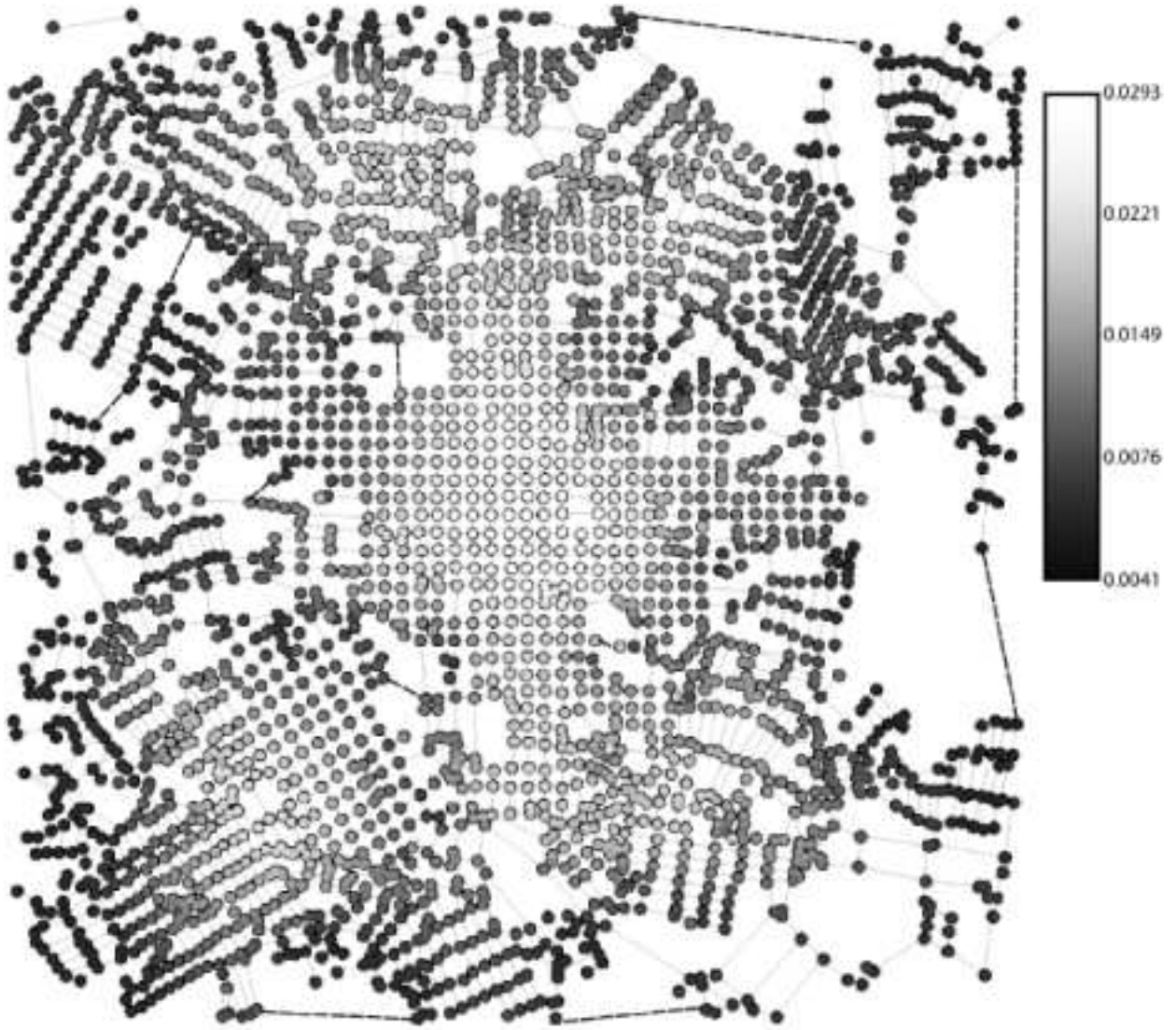


FIG. 1: Network of urban streets of São Carlos, Brazil. The gray level of the nodes indicates their respective averaged outward accessibility according to the legend at the right-hand side. The dashed lines represent the hypothetical additional edges aimed at improving the accessibility.

After a real-world structure has been properly represented as a complex network, a diversity of measures can be obtained, ranging from simple features such as the node degree and clustering coefficient, to more sophisticated such as shortest path lengths and betweenness centrality. These measures have allowed the comprehensive description and characterization of several complex systems [3, 4].

The *transition probability* that a moving agent node reaches a node  $j$  after departing from a node  $i$ , through a self-avoiding walk after  $h$  steps, is henceforth expressed as  $P_h(i, j)$ . In order to estimate this probability, a total of  $M$  self-avoiding *random* walks, starting from the node  $i$  and proceeding  $S$  steps, are performed. Note that these walks stop when one of the following three conditions is met: (i) the walk reaches the maximum pre-defined value

of steps ( $S + 1$  nodes); (ii) the walk reaches an extremity node, i.e., a node with degree one; or (iii) the walk cannot proceed further because all of immediate neighbors of the node at step  $h$  were already visited. The probability  $P_h(i, j)$  can then be estimated in terms of the number of times that the walks departing from a node  $i$  reach the node  $j$  after  $h$  steps, divided by the number of walks,  $M$ . Note that  $P_h(i, j)$  is typically different of  $P_h(j, i)$ . After the probabilities are estimated for each node, it is possible to calculate the *diversity entropy signature*  $E_h(i)$  of a node  $i$  after  $h$  steps as [20]:

$$E_h(i) = - \sum_{j=1}^N \begin{cases} 0 & \text{if } P_h(i, j) = 0 \\ P_h(i, j) \ln(P_h(i, j)) & \text{if } P_h(i, j) \neq 0 \end{cases} \quad (1)$$

Although the diversity entropy provides an interesting

quantification of the accessibility of the nodes, the *outward accessibility* has been proposed as a normalization of the *diversity entropy*, allowing direct comparison with other measures regarding non-linear transient dynamics of the network (e.g. *inward accessibility* [19]). The outward accessibility  $OA_h(i)$  of a node  $i$  after  $h$  steps can be immediately calculated from the diversity entropy as

$$OA_h(i) = \frac{\exp(E_h(i))}{N - 1} \quad (2)$$

The outward accessibility was estimated for the central part of the urban streets of São Carlos, which is represented by a network with  $N = 2812$  nodes and  $E = 4713$  edges (Fig. 1). The total length of the self-avoiding random walks performed for each node was  $S = 60$ , and 10.000 walks were performed so as to obtain accuracy in probability estimations. In Fig. 1 the gray levels of the nodes corresponds to their respective accessibility, averaged over all the steps. An interesting result which is evident in this figure is that most part of the highly accessible nodes corresponds to the downtown São Carlos, located at the central region of the map. Another important property is the high spatial discriminative power provided by the outward accessibility measurement: it can be clearly seen from Fig. 1 that the nodes situated at the border of the network have the smallest outward values, while the inner nodes have the highest outward values. Interestingly, nodes with low outward accessibil-

ity can be found even downtown. It is interesting to recall that the ability of the accessibility approach to identify the most central (against the borders) parts of a network is not restricted to geographical networks, but can be immediately applied to any other type of complex network.

In order gain more insights regarding the accessibility of the different parts of the town, we also considered hypothetical new edges (i.e., new streets) connecting some nodes of the periphery and internal regions of the town (represented as dashed lines in Fig. 1). This new arrangement allowed a study of the potential impact of the new edges in the accessibility of their neighborhood. The mean accessibility was computed considering the nodes located up to seven blocks away from the nodes that received the new connections. Figure 2 shows these values for the original network and for the enhanced network. Note an increase of 21% in the accessibility after approximately  $h = 15$  steps for the place where the new edges were added. This result shows that major improvements of accessibility can be achieved by adding just a few streets at strategic locations.

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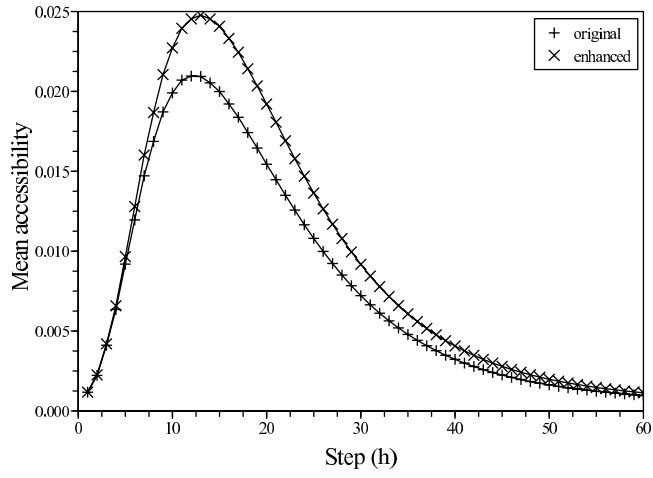


FIG. 2: Mean accessibility of the original network and the enhanced network. In both cases, only the nodes with up to seven edges of distance from the nodes that received the new connections were taken into account. Note the substantial increase in the accessibility as a consequence of the new connections.